

Influence of inelastic relaxation time on intrinsic spin Hall effects in a disordered two-dimensional electron gas

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The influence of inelastic relaxation time on the intrinsic spin Hall effects in a disordered two-dimensional electron gas with Rashba interaction is studied, which clarifies the controversy of impurity effects in the system. We reveal that, due to the existence of inelastic scattering, the spin Hall conductivity does not vanish when the impurity concentration diminishes to zero no matter it is non-magnetically or magnetically disordered. The spin accumulation is evaluated by using the obtained spin Hall conductivity, and an alternate route is suggested to verify the intrinsic spin Hall effect by measuring the spin accumulation at different ratios.

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I. INTRODUCTION

Much attention has been paid to the study of intrinsic spin Hall effects (ISHE) which is expected to bring in practical applications in spintronics. In the intrinsic spin Hall effect, the spin current arises from the spin-orbit-dependent band structure. Theoretically, ISHE may exist in the p-type semiconductor [1] and two-dimensional electron gas (2DEG) [2]. After Sinova et al. [2] predicted a universal spin Hall conductivity, $\sigma_{sH} = e/8\pi$ in clean 2DEG, several groups [3, 4, 5, 6, 7] indicated that an arbitrarily small impurity concentration would suppress the spin Hall conductivity to zero due to the vertex corrections. Rashba [6] and Dimitrova [7] proved that the spin-Hall current was always zero in the non-magnetically disordered system while Grimaldi et al. [8] and Krotkov [9] noticed that the spin Hall conductivity is not zero unless it is in the special situation for a large Fermi circle, quadratic band structure and momentum-independent Rashba coefficient. Very recently, Inoue et al. [10] and Wang et al. [11] recognized that the spin Hall conductivity is nonzero in the presence of magnetic impurities. However, no one can explain why there exists a discontinuous jump in σ_{sH} between the clean limit and a clean system. Thus there remains a puzzle that why the clean limit of the spin Hall conductivity does not equal to the one in a clean system. Opposite to analytical results, a numerical calculation in a finite 2DEG by Nomura et. al. [12] manifested a robust spin Hall conductivity that falls to zero only when the inverse of the elastic relaxation time (elastic lifetime in brevity) is larger than the spin-orbit splitting of the bands, i.e., $1/\tau > \Delta$. This raises a question that why there is such a controversy between the numerical calculation and the analytical consequences. A consistent comprehension of the aforementioned issues becomes an obligatory issue.

It is known that all the states in two dimensional infinite system in the presence of disorder are localized [13] at zero temperature. At finite temperature, there exist delocalized states because of the presence of dephasing. The interference occurs only inside the decoherence

length so that the electronic conductivity depends on the ratio of elastic to inelastic lifetimes. Although the importance of dephasing in the electrical charge transport in 2D systems has been addressed in the weak localization theory [13, 14, 15, 16, 17, 18, 19], there has been no discussions on the role of dephasing in the spin Hall effect. We will show in the present paper that the dephasing plays a crucial role in the spin Hall effect, leading to a nonzero conductivity for an arbitrary impurity concentration.

This paper is organized as follows. In next section, we give a description of the system and make a general formulation by introducing inelastic relaxation time. In section III, we investigate the influence on spin Hall conductivity by nonmagnetic impurities. In section IV, we consider magnetic impurities and study their effects on the spin Hall effect. In section V, we plot the curves of the conductivity versus the ratio of elastic to inelastic relaxation times and observe their asymptotic behaviors. We also discuss the corresponding finite temperature behaviors. In section VI, we evaluate the spin accumulation in terms of the spin Hall conductivity we obtained. Our conclusive remarks are briefly summarized in the last section.

II. GENERAL CONSIDERATION

The Hamiltonian for a 2DEG with Rashba spin-orbit coupling is given by $H = p^2/2m^* + \alpha(\sigma^x p_y - \sigma^y p_x) + V_{dis}$ with V_{dis} denoting potentials produced by either non-magnetic impurities or magnetic impurities. The spin current is defined as

$$\begin{aligned} J_y^z(\mathbf{p}) &= \frac{1}{4}(v_y \sigma_z + \sigma_z v_y) \\ &= \frac{1}{2} \frac{p_y}{m^*} \sigma_z. \end{aligned} \quad (1)$$

On the basis of Kubo's formalism, the spin Hall conductivity in response to a dc electric field at zero temperature

can be expressed as

$$\sigma_{sH} = -\frac{e}{2\pi} \text{Tr}[J_y^z G^r(\mu) j_x G^a(\mu)], \quad (2)$$

where $j_x = \frac{d}{dp_x} \left(\frac{p^2}{2m^*} + \alpha(\sigma^x p_y - \sigma^y p_x) \right)$ is the electrical current; G^r and G^a denote the retarded and advanced Green's functions, respectively. The trace is taken over momentum and spin indices.

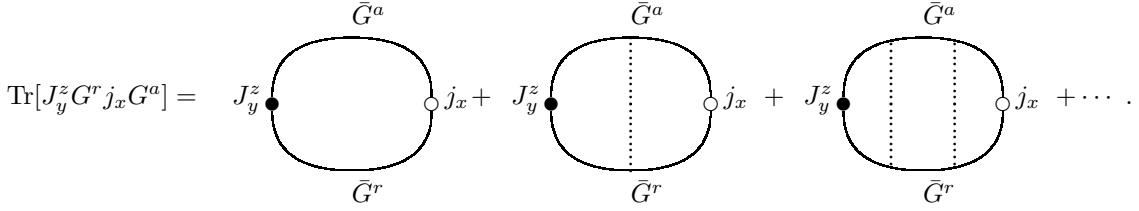
The unperturbed Hamiltonian $H_0 = p^2/2m^* + \alpha(\sigma^x p_y - \sigma^y p_x)$ can be diagonalized to be $\varepsilon_{\pm}(p) = p^2/2m^* \mp p\alpha$ by the unitary matrix,

$$U(\mathbf{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ ie^{i\varphi_{\mathbf{p}}} & -ie^{i\varphi_{\mathbf{p}}} \end{pmatrix},$$

where $\varphi_{\mathbf{p}}$ refers to the azimuthal angle of the momentum \mathbf{p} with respect to the x axis. Then the free particle Green's function in chiral bases can be expressed as

$$G_{0(ch)}^r(\mathbf{p}, \mu) = \begin{pmatrix} \frac{1}{\mu - \varepsilon_+ + i\eta} & 0 \\ 0 & \frac{1}{\mu - \varepsilon_- + i\eta} \end{pmatrix}. \quad (3)$$

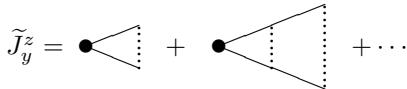
We employ the diagrammatic technique [11] to calculate the average spin Hall conductivity over the distribution of impurities. The trace in Eq.(2) is expanded as a sum of diagrams:



The spin Hall conductivity consists of two parts σ_{sH}^0 and σ_{sH}^L , namely,

$$\begin{aligned} \sigma_{sH}^0 &= -\frac{e}{2\pi} \text{Tr}[J_y^z \bar{G}^r(\mu) j_x \bar{G}^a(\mu)], \\ \sigma_{sH}^L &= -\frac{e}{2\pi} \text{Tr}[\tilde{J}_y^z \bar{G}^r(\mu) j_x \bar{G}^a(\mu)], \end{aligned} \quad (4)$$

where the former represents the contribution of one-loop diagram while the latter arises from the vertex corrections. Here \tilde{J}_y^z refers to the corrected-spin-current vertex which obeys a self-consistent equation [11] illustrated by the following diagram,



Solving the self-consistent Born's equation, one obtain the Green's function in the presence of impurities:

$$\bar{G}_{(ch)}^r(\mathbf{p}, \mu) = \begin{pmatrix} \frac{1}{\mu - \varepsilon_+ + \frac{i}{2\tau}} & 0 \\ 0 & \frac{1}{\mu - \varepsilon_- + \frac{i}{2\tau}} \end{pmatrix}, \quad (5)$$

where the momentum-relaxation time τ is related to the impurity concentration n_{im} and scattering strength u ,

namely $1/\tau = n_{\text{im}} u^2 m^*$ whatever for nonmagnetic or magnetic impurities.

To uncover the puzzle and disentwine the controversy in the impurity effects on the spin Hall conductivity, we need look through the features of an infinite 2-dimensional quantum system. An infinite system with infinitesimal impurity concentration contains infinite number of impurities. Diluting the impurity concentration means to increase the distance between impurities. The infinite quantum systems with different impurity concentrations can be mapped into each other by redefining the length scale and Fermi wavelength, whereas they cannot be directly mapped into a clean system. This means that the clean limit of an infinite quantum system is actually not a clean system if the effect of dephasing is ignored! The dephasing can be characterized by inelastic relaxation time.

We employ an imaginary self-energy to represent the inelastic scattering, which was early introduced in the weak localization theory [16, 19]. Thus the Green's functions $\bar{G}^r(\mu)$ and $\bar{G}^a(\mu)$ are substituted by frequency-dependent functions $\bar{G}^r(\mu + \omega/2)$ and $\bar{G}^a(\mu - \omega/2)$ where ω is replaced by i/τ_i with τ_i being the inelastic relaxation time (inelastic lifetime in brevity). As a result, the aver-

aged Green's function in chiral bases turns to be

$$\bar{G}_{(ch)}^r = \begin{pmatrix} \frac{1}{\mu - \varepsilon_+ + \frac{i}{2\tau} + \frac{i}{2\tau_i}} & 0 \\ 0 & \frac{1}{\mu - \varepsilon_- + \frac{i}{2\tau} + \frac{i}{2\tau_i}} \end{pmatrix}. \quad (6)$$

In the limit case $\tau_i \rightarrow \infty$, the above Green's function gives rise to Eq. (5).

III. FOR NONMAGNETIC IMPURITIES

We consider a system in the presence of nonmagnetic impurities. The interaction between electron and impurities is expressed as

$$V_{dis} = \sum_{i=1}^N \int d\mathbf{r}^2 u \delta(\mathbf{r} - \mathbf{R}_i) \hat{\psi}^\dagger(\mathbf{r}) \hat{\psi}(\mathbf{r}). \quad (7)$$

The calculation of the momentum-integral of the product of Green's functions is carried out analytically. For simplicity, we adopt the limit of large Fermi energy $\mu \rightarrow \infty$ firstly. The σ_{sH}^0 we obtained remains the so called universal value $e/8\pi$.

Furthermore, we calculate the vertex correction. The self-consistent equation for \tilde{J}_y^z reads

$$\tilde{J}_y^z = \frac{n_i u^2}{V} \sum_{\mathbf{p}} \bar{G}^a(\mathbf{p}, 0) (J_y^z(\mathbf{p}) + \tilde{J}_y^z) \bar{G}^r(\mathbf{p}, 0). \quad (8)$$

Solving the corrected-spin-current vertex, we get the corresponding matrix element, saying

$$(\tilde{J}_y^z)_{\uparrow\downarrow} = \frac{-i}{4\alpha m^*(2\tau - \tau')}, \quad (9)$$

where $1/\tau' = 1/\tau + 1/\tau_i$. Then we obtain the vertex correction

$$\sigma_{sH}^L = \frac{-ie(\tilde{J}_y^z)_{\uparrow\downarrow} m^* \alpha \tau'}{2\pi} = \frac{-e}{8\pi(2\frac{\tau}{\tau'} - 1)}. \quad (10)$$

As a reasonable asymptotic behavior, the corrected-spin-current vertex tends to zero when $\tau \rightarrow \infty$ which is the realistic clean limit. In our formulation, the momentum-integral of the product of Green's functions $\sum_{\mathbf{p}} \bar{G}^r(\mu, \mathbf{p}) j_x(\mathbf{p}) \bar{G}^a(\mu, \mathbf{p})$ is convergent when $\tau \rightarrow \infty$ for a finite τ_i such that the vertex correction goes to zero in the clean limit. It is due to the divergence of the momentum-integral when $1/\tau_i = 0$ in current literatures that the vertex correction is not zero in the clean limit. The vertex correction arises from the interference of the scattering waves from different impurities. When

the elastic lifetime is much larger than the inelastic lifetime or the distance between impurities is much larger than the decoherence length, the interference effect will disappear. So it is natural that the vertex correction goes to zero in the clean limit, removing the discontinuity in the conductivity variation.

The total spin Hall conductivity is then given by

$$\sigma_{sH} = \frac{e}{8\pi} \frac{1}{1 + \frac{\tau_i}{2\tau}}, \quad (11)$$

which fulfills $\lim_{\tau \rightarrow \infty} \sigma_{sH} = e/8\pi$ for a finite inelastic lifetime. The clean limit of the spin Hall conductivity is precisely the conductivity in a clean 2DEG due to the disappearance of vertex corrections. Here we do not consider the finite-size effect since the system's size is assumed to be much larger than the mean-free-path or the decoherence length.

In the limit $\tau_i \rightarrow \infty$, we have $\sigma_{sH} = 0$ in coincidence with the result for a system without inelastic scattering. For large τ/τ_i , the spin transport belongs to the semiclassical regime and the conductivity goes to $e/8\pi$. When τ/τ_i is small, the transport falls into the quantum regime and the conductivity tends to zero. Our result demonstrates the difference between the semiclassical and quantum transport regimes in the spin Hall effect. It was proved that the spin Hall conductivity vanishes when there exist nonmagnetic impurities in a homogenous system [6, 7]. It was also indicated [6] that inhomogeneities facilitate spin currents. However, their conclusions are not valid after taking account of inelastic lifetime because their Schrödinger equation is not appropriate to describe a system with inelastic scattering. Our consequence is that the spin Hall conductivity is not zero in a realistic system.

In calculating the momentum-integral of Green's functions in the above, we assumed an infinite Fermi energy for simplicity. For a finite Fermi energy, we can also evaluate the spin Hall conductivity in semiclassical approximation [7, 11]. We get formally a similar σ_{sH}^0 as in Ref. [7] with the only difference that τ becomes τ' ,

$$\sigma_{sH}^0 = \frac{e}{8\pi} (1 - \frac{1}{1 + (\Delta\tau')^2}). \quad (12)$$

Now the matrix element of the corrected-spin-current vertex is found to be

$$(\tilde{J}_y^z)_{\uparrow\downarrow} = \frac{-iv_F \Delta \tau'^2}{4\tau(1 + (\Delta\tau')^2) - 2\tau'(1 + (\Delta\tau')^2) - 2\tau'}, \quad (13)$$

where Δ is the spin-orbit splitting at the Fermi surface, and v_F the Fermi velocity. The vertex correction is obtained as

$$\begin{aligned} \sigma_{sH}^L = & \frac{e}{8\pi} \frac{(\Delta\tau')^2}{1 + (\Delta\tau')^2} \times \\ & \frac{-\Delta^2 \tau'^3}{2\tau(1 + (\Delta\tau')^2) - \tau'(1 + (\Delta\tau')^2) - \tau'}. \end{aligned} \quad (14)$$

Consequently, the total spin Hall conductivity is given by

$$\sigma_{sH} = \frac{e}{8\pi} \frac{(\tau_i \Delta)^2}{(1 + \tau_i/\tau)^2 + (\tau_i \Delta)^2 (\tau_i/2\tau + 1)}. \quad (15)$$

The vertex correction still goes to zero in the clean limit, leading to continuous change of the conductivity with respect to impurity concentration. For a finite Fermi energy, the spin Hall conductivity depends not only on the ratio of elastic to inelastic lifetimes but also on the spin-orbit splitting at the Fermi surface. The clean limit of Eq. (15) is $\frac{e}{8\pi} \frac{(\tau_i \Delta)^2}{1 + (\tau_i \Delta)^2}$, depending on the inelastic lifetime τ_i . If the inelastic lifetime becomes infinitely long, Eq. (15) diminishes to zero, recovering the result we are familiar with.

Let us compare with the numerical result [12]. Nomura et al. set a finite η^{-1} to guarantee the convergence in their numerical calculation and found the spin Hall conductivity increases as the impurity concentration decreases and $\eta\tau$ increases. In their paper η^{-1} is called the electric field turn-on time, in fact, it should be the inelastic lifetime τ_i . Since $\eta\tau$ corresponds to the ratio τ/τ_i , their result supports our present conclusion.

IV. FOR MAGNETIC IMPURITIES

Very recently, the spin Hall conductivities in the presence of magnetic impurities are calculated in Refs. [10, 11] manifesting the spin transport properties of magnetically and non-magnetically disordered systems are different. It becomes an obliged task to confirm whether this difference remains when the inelastic scattering is not ignored. We adopt the scattering potentials of magnetic impurities as

$$V_{dis} = \sum_{i=1}^N \int d\mathbf{r}^2 u \delta(\mathbf{r} - \mathbf{R}_i) \times \hat{\psi}^\dagger(\mathbf{r}) \begin{pmatrix} \cos \theta_i & \sin \theta_i e^{-i\phi_i} \\ \sin \theta_i e^{i\phi_i} & -\cos \theta_i \end{pmatrix} \hat{\psi}(\mathbf{r}). \quad (16)$$

We firstly consider the infinite Fermi energy limit, obtaining $\sigma_{sH}^{M0} = e/8\pi$. The self-consistent equation of corrected vertex is expressed as

$$\tilde{J}_y^{Mz} = \frac{n_i u^2}{V} \sum_{\mathbf{p}} \int d\theta d\phi \frac{1}{4\pi} \sin \theta \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix} \times \bar{G}^a(\mathbf{p}, 0) (J_y^z(\mathbf{p}) + \tilde{J}_y^{Mz}) \bar{G}^r(\mathbf{p}, 0) \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix}. \quad (17)$$

The matrix element of corrected vertex is found to be

$$(\tilde{J}_y^{Mz})_{\uparrow\downarrow} = \frac{i}{4\alpha m^*(6\tau + \tau')}. \quad (18)$$

And the vertex correction is obtained

$$\sigma_{sH}^{ML} = \frac{e}{8\pi} \frac{1}{6\frac{\tau}{\tau'} + 1}. \quad (19)$$

Thus the spin Hall conductivity is given by

$$\sigma_{sH}^M = \frac{e}{8\pi} \left(1 + \frac{1}{6\frac{\tau}{\tau_i} + 7} \right). \quad (20)$$

Obviously, the clean limit, $\tau \rightarrow \infty$, of the spin Hall conductivity in magnetically disordered system is also the universal value $e/8\pi$ [2]. In the limit of infinite inelastic lifetime, σ_{sH}^M goes to $e/7\pi$ in consistent with the recent results [10, 11] for magnetic impurities.

We also consider the case for a finite Fermi energy. The σ_{sH}^{M0} here keeps the same result as Eq. (12) for non-magnetic impurities. While the matrix element of the corrected vertex reads

$$(\tilde{J}_y^{Mz})_{\uparrow\downarrow} = \frac{iv_F \Delta \tau'^2}{12\tau(1 + (\Delta\tau')^2) + 2\tau'(1 + (\Delta\tau')^2) + 2\tau'} \quad (21)$$

and vertex correction is given by

$$\sigma_{sH}^{ML} = \frac{e}{8\pi} \frac{(\Delta\tau')^2}{1 + (\Delta\tau')^2} \times \frac{\Delta^2 \tau'^3}{6\tau(1 + (\Delta\tau')^2) + \tau'(1 + (\Delta\tau')^2) + \tau'}. \quad (22)$$

The total spin Hall conductivity in the presence of magnetic impurities is then expressed as

$$\sigma_{sH}^M = \frac{e}{8\pi} \frac{(\tau_i \Delta)^2}{(1 + \tau_i/\tau)^2 + (\tau_i \Delta)^2} \frac{7 + 6\tau/\tau_i}{8 + 6\tau/\tau_i}. \quad (23)$$

When $\tau/\tau_i = 0$, Eq. (23) becomes $\frac{e}{8\pi} \frac{8(\Delta\tau)^2}{8 + 7(\Delta\tau)^2}$ recovering the result of our recent paper [11]. When $\tau/\tau_i \rightarrow \infty$, Eq. (23) reduces to $\frac{e}{8\pi} \frac{(\tau_i \Delta)^2}{1 + (\tau_i \Delta)^2}$ which is precisely the conductivity in the clean limit of non-magnetic impurities for finite Fermi energy.

V. DISCUSSION ON THE IMPURITY EFFECTS

The above studies exhibited that the spin Hall conductivities, in the presence of either nonmagnetic or magnetic impurities, depend on the ratio of elastic to inelastic lifetimes. We plot the curves of the conductivity versus τ/τ_i in the presence of nonmagnetic and magnetic impurities respectively in Fig. 1. As the impurity concentration decreases (i.e., τ increases), σ_{sH}^M decreases while σ_{sH} increases monotonously. The magnetic impurities

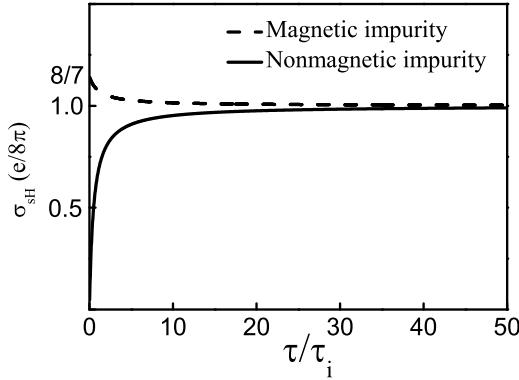


FIG. 1: Spin Hall conductivity versus the ratio of elastic to inelastic lifetimes is plotted in both cases for nonmagnetic and magnetic impurities.

enhance ISHE, while the nonmagnetic impurities suppress it. In the clean limit, both magnitudes approach to $e/8\pi$. In the dirty limit or infinite inelastic-life-time limit σ_{sH}^M goes to $e/7\pi$ but σ_{sH} goes to zero.

It is worthwhile to consider the finite-temperature case. We have already shown that the spin Hall conductivity depends on the ratio of elastic to inelastic lifetimes merely in the approximation of infinitely wide band and infinitely large Fermi energy. These two characteristic times are important to both the charge and spin transport properties for 2D systems. Our results provide a clue to compare those characteristic times by making use of the charge and spin transport experiments. At low temperature, the elastic lifetime τ is determined by the impurity concentration and is a constant independent of the temperature, while the inelastic lifetime τ_i relates to the electron-electron interaction and decreases when the temperature increases. In general, $\tau_i \propto T^{-p}$ whose exponent p depends on the scattering mechanism (e.g., $p = 2$ for electron-electron scattering), then the spin Hall conductivity is proportional to $1/(1 + KT^{-p})$ increasing as the temperature raising. Since the spin Hall conductivity is sensitive to the temperature, its temperature dependence is expected to distinguish ISHE and ESHE.

VI. SPIN ACCUMULATION

The spin accumulation brought about by spin Hall effect has been observed [20, 21, 22, 23, 24, 25] in experiments. We therefore evaluate the spin accumulation generated by intrinsic spin Hall current based on the results we obtained in the above. We consider a bar of width W in y -direction with an applied electric field along x -direction, for which a spin current along y -direction will take place. If the width is much larger than the mean-free-path and the decoherence length $W \gg l, l_\varphi$, the sys-

tem is in the diffusive transport regime. The spin accumulation can be studied by the diffusion equation

$$D \frac{d^2 S_z}{dy^2} = \frac{S_z}{\tau_s}, \quad (24)$$

where $D = v_F^2 \tau / 2$ is the diffusive coefficient, τ_s the spin relaxation time (spin lifetime in brevity), and $S_z = 1/2(n_\uparrow - n_\downarrow)$. The spins accumulate at the edges of the bar until the diffusive spin current in opposite direction balances the spin Hall current. Then a steady spin distribution is established. For simplicity, we do not take the influence of the spin accumulation on the spin Hall current into account. The boundary condition is then given by

$$J_y^z = D \frac{dS_z}{dy} \Big|_{y=\pm W/2}. \quad (25)$$

The solution of the above diffusion equation is

$$S_z(y) = \frac{J_y^z \sqrt{\tau_s / 2D}}{W} \sinh \frac{y}{\sqrt{D\tau_s / 2}}. \quad (26)$$

Eq. (26) is a good approximation for the spin distribution in the region away from the edge. Approaching the edge, however, the spin accumulation will affect the spin current significantly, leading to a dramatic loss of spin current. According to Eq. (26), the spin density at the edge of the sample is

$$S_z = \frac{eE}{8\pi v_F} \frac{2\tau/\tau_i}{2\tau/\tau_i + 1} \sqrt{\frac{\tau_s}{\tau}} \tanh \frac{W}{v_F \sqrt{\tau\tau_s}} \quad (27)$$

in the presence of non-magnetic impurities, and

$$S_z = \frac{eE}{8\pi v_F} \frac{6\tau/\tau_i + 8}{6\tau/\tau_i + 7} \sqrt{\frac{\tau_s}{\tau}} \tanh \frac{W}{v_F \sqrt{\tau\tau_s}} \quad (28)$$

in the presence of magnetic impurities. Here E stands for the electric field in x -direction. The spin accumulation depends on the three characteristic times: the elastic lifetime, the inelastic lifetime and the spin lifetime. Because the different characteristic times can be manipulated separately, our result proposes an alternate route to verify the ISHE by measuring the spin Hall accumulation under different conditions.

The spin accumulation due to spin Hall effect was analyzed theoretically [25, 26, 27, 28]. In 2DEG, the spin accumulation was observed in Ref. [24] where the authors regarded it arising from ESHE because the relation $\Delta < 1/\tau$ is satisfied in their experiments. In this situation, they believe the ISHE does not exist according to Nomura's calculation [12]. But our result shows that the spin Hall conductivity is not zero, instead, it depends on the characteristic times of the system, especially on the ratio of elastic to inelastic lifetimes. Whether the spin accumulation observed in experiment comes from ESHE or ISHE is worthy analyzed more carefully. An experiment at different temperature is expected to be helpful.

VII. CONCLUSIVE REMARKS

We indicated that the clean limit of an infinite quantum system is not a clean system if the effect of dephasing is ignored. It was hence natural for one to have obtained a discontinuity (zero with arbitrarily small impurity concentration but $e/8\pi$ with no impurities) in the spin Hall conductivity for infinite systems without taking account of dephasing which is characterized by inelastic relaxation time. The disagreements between numerical result and the others' analytical results become inevitable because the former [12] dealt with a finite system and the clean limit of a finite system is a clean system.

We exposed the influence of inelastic relaxation time to ISHE for 2D electrons in the presence of magnetic and nonmagnetic impurities, respectively. We found that the inelastic scattering plays an important role in the spin Hall effect, leading to a nonzero spin Hall conductivity for arbitrary impurity concentrations. In the dirty limit, the spin Hall conductivity goes to zero and $e/7\pi$ for non-

magnetic or magnetic impurities, respectively. It tends to $e/8\pi$ in the clean limit regardless of magnetically or non-magnetically disordered systems. We revealed the importance of characteristic times, such as the elastic, inelastic and spin lifetimes for ISHE. The spin Hall conductivity is shown to depend on the ratio of elastic to inelastic lifetime and varies when temperature changes, which provides a method to distinguish the ISHE and ESHE by measuring the spin current at different temperatures. Based on the spin Hall conductivity we obtained, we evaluated the spin accumulation and presented an alternate route to verify the ISHE by measuring it under different conditions.

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